Circular Optical Nanoantennas - An Analytical Theory

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Subject and Motivation

• antennas for visible light now feasible 1,2
• metals no perfect conductors in this spectral domain
• goal: understand nanoantenna characteristics on analytical grounds - self consistent predictions of supported plasmonic eigenmodes requires three ingredients
• profile of plasmonic field, dispersion relation and reflection coefficient to calculate resonances using a Fabry-Perot model 3
• circular nanoantennas
• axial symmetry
• properties tunable via stack composition

A Simple Resonator Model

• Hankel functions diverge at origin - cannot be eigenmodes
• stationary solutions in given symmetry: Bessel functions - field inside:

\[ E_{mn}^{-r}(\rho, z) = J_m(k_{BSP}\rho) \cdot \alpha(z) \]
• apparent length change due to phase of reflection
• Fabry-Perot resonance condition:

\[ 2 \cdot k_{BSP} R_{m} + \phi_m = 2 \cdot \pi \cdot n(J_m) \]

A plane wave excites a dipolar plasmonic Bessel-resonance of an 80 nm thick metallic disc. The reflection phase leads to an apparent length change.

Theory of Hankel Reflection

• Hankel plasmons propagate outwards across the resonator and get reflected
• composition of the stack fixes dispersion relation and mode profile \( a(z) \)
• neglecting reflection into other modes; field representation (inner, outer):

\[ E_{mn}^{\text{in}}(\rho, z) = \mathbb{A}_m(k_{BSP}\rho) \cdot a(z) \quad \text{with} \quad \mathbb{A}_m(k_{BSP}\rho) = H_{mn}^{-1}(k_{BSP}\rho) + r_m \cdot H_{mn}^{0}(k_{BSP}\rho) \]

\[ E_{mn}^{\text{out}}(r) = \int_{-\infty}^{\infty} c_m(k_{BSP}\rho) H_{mn}^{1}(k_{BSP}\rho) e^{-ik_{BSP}\rho} \]

• ansatz: continuity of \( H_\rho \) and \( \int E_\rho \cdot H_\rho dz \) at \( \rho = R \) to derive the reflection coefficient

\[ r_m = \frac{2\pi \epsilon_0 k_{BSP} \sigma H_{mn}^{1}(k_{BSP}R) - DH_{mn}^{0}(k_{BSP}R)}{-2\pi \epsilon_0 k_{BSP} \sigma H_{mn}^{1}(k_{BSP}R) + DH_{mn}^{0}(k_{BSP}R)} \]

with the abbreviations

\[ L_m = \int_{-\infty}^{\infty} \frac{H_{mn}^{1}(k_{BSP}z) \cdot \sigma dz}{DH_{mn}^{0}(k_{BSP}z) - k_{BSP}^2} \quad \text{and} \quad DH_{mn}^{0}(z) = \frac{\partial, H_{mn}^{1}(z), \sigma}{\partial \int_{-\infty}^{\infty} \epsilon(z) a(z)^3 dz} \],

\[ B^m(k) = \int_{-\infty}^{\infty} \epsilon(z) a(z) e^{-ikz} dz \]

How to use this results to understand the characteristics of circular nanoantennas?

Field Profile and Scaling

• resonator model implies form of the field and scaling of resonant radii
• educated guesses; have to be verified: simplest structure: a metallic disc
• numerical simulations: plane wave excitation at \( \nu = 625 \) THz

• a) \& c) field profile
  • 80 nm thick disc, excitation of even mode
  • the electric field shows a qualitative agreement to a Bessel type plasmonic field
• b) scaling
  • resonant radii for thicknesses from 6 nm to 160 nm are linearly related to the roots of \( J_1 \)

Spectral Predictions

• resonances of a silver disc, \( R = 900 \) nm
• a) spectrum with predicted resonances
• b) \& c) actual field distributions for 4th and 5th resonance
• SPP highly damped for 5th order - deviation from Bessel form, other effects important
• also spectral very good agreement to numerics within limitations of theory

Limiting Cases

• A: verification of known results 4 in infinite disc limit for several orders
• B: even and odd modes converge to same result for increasing thickness

Conclusions

• theory for radially propagating Hankel-type SPPs in piecewise homogeneous circular nanoantennas
• properties explained by Fabry-Perot model using phase of reflection in agreement to simulations
• antenna properties tunable via stack composition

References

3 T. H. Tamir et al. Optical Nanomodulated Apertures as Cavities for Dipolar Emitters, Nano Lett. 11, 3028 (2011)