Circular Optical Nanoantennas - An Analytical Theory

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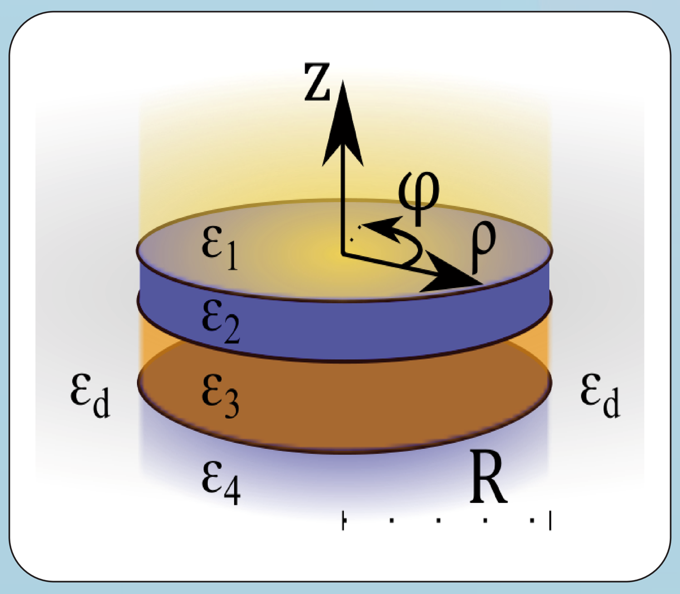
Subject and Motivation

- **antennas for visible light** now **feasible** [1,2]
- **metals no perfect conductors** in this spectral domain
- **goal**: understand nanoantenna characteristics on **analytical** grounds - self consistent predictions of supported **plasmonic eigenmodes** requires three ingredients
  - **profile** of plasmonic field, **dispersion relation** and
  - **reflection coefficient** to calculate resonances using a Fabry-Perot model [3]
- **circular nanoantennas**
  - **axial symmetry**
  - **properties tunable** via stack composition



Theory of Hankel Reflection

- **Hankel plasmons** propagate outwards accross the resonator and get **reflected**
  - **composition** of the stack fixes **dispersion relation** and **mode profile** $a(z)$
  - neglecting reflection into other modes; **field representation** (inner, outer):

$$E_z^{m,-}(\rho,z) = \mathcal{A}_m(k_{SPP}\rho)\cdot a(z) \text{ with}$$
$$\mathcal{A}_m(k_{SPP}\rho) = H_m^1(k_{SPP}\rho) + r_m \cdot H_m^2(k_{SPP}\rho)$$
$$E_z^{m,+}(\mathbf{r}) = \int_{-\infty}^{\infty} c_m(k_z)\, H_m^1\left(\sqrt{\epsilon_d k_0^2 - k_z^2}\rho\right) e^{\mathrm{i}k_z z} dk_z$$

- **ansatz**: continuity of H_φ and $\int E_z \cdot H_\varphi dz$ at $\rho = R$ to derive the **reflection coefficient**

$$r_m = \frac{2\pi\epsilon_d\, k_{SPP}\sigma\, H_m^1(k_{SPP}R) - DH_m^1(k_{SPP}R)\, I_m}{-2\pi\epsilon_d\, k_{SPP}\sigma\, H_m^2(k_{SPP}R) + DH_m^2(k_{SPP}R)\, I_m}$$

with the abbreviations

$$I_m \equiv \int_{-\infty}^{\infty} \frac{H_m^1\left(\sqrt{\epsilon_d k_0^2 - k_z^2}R\right)}{DH_m^1\left(\sqrt{\epsilon_d k_0^2 - k_z^2}R\right)} \cdot\sqrt{\epsilon_d k_0^2 - k_z^2}\cdot B^-(k_z)\cdot B^+(k_z)\, dk_z$$

and

$$DH_m^{1/2}(x) \equiv \partial_x H_m^{1/2}(x)\ ,\ \sigma \equiv \int_{-\infty}^{\infty} \epsilon(z)\, a(z)^2\, dz\ ,$$
$$B^{\pm}(k) \equiv \int_{-\infty}^{\infty} \epsilon(z)\, a(z)\, e^{\pm\mathrm{i}kz} dz$$

How to use this results to understand the characteristics of circular nanoantennas?

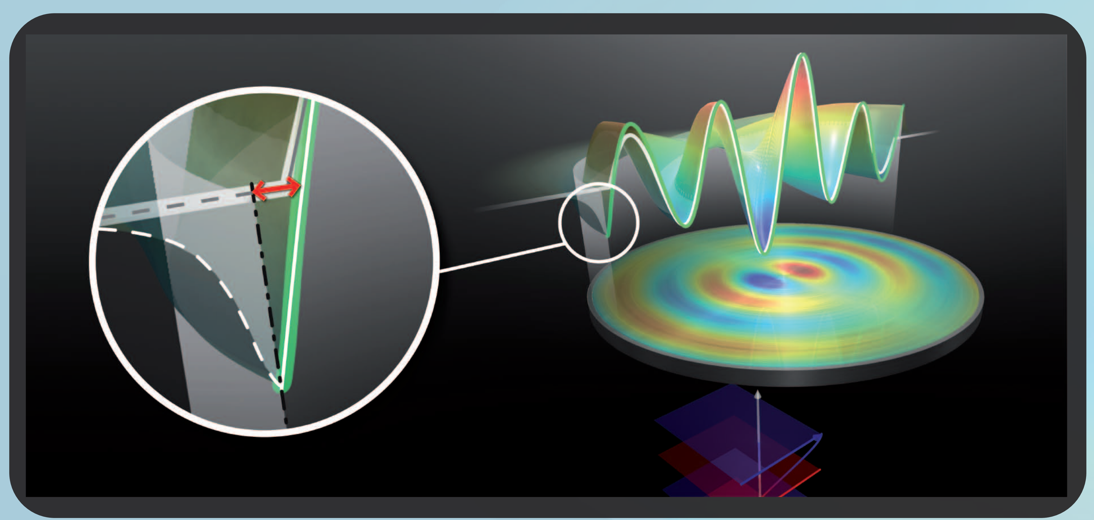
A Simple Resonator Model

- **Hankel functions diverge** at origin - cannot be eigenmodes
  - stationary solutions in given symmetry: **Bessel functions** - field inside:

$$E_z^{m,-}(\rho,z) = J_m(k_{SPP}\rho)\cdot a(z)$$

- **apparant length change** due to phase of reflection
- Fabry-Perot **resonance condition**:

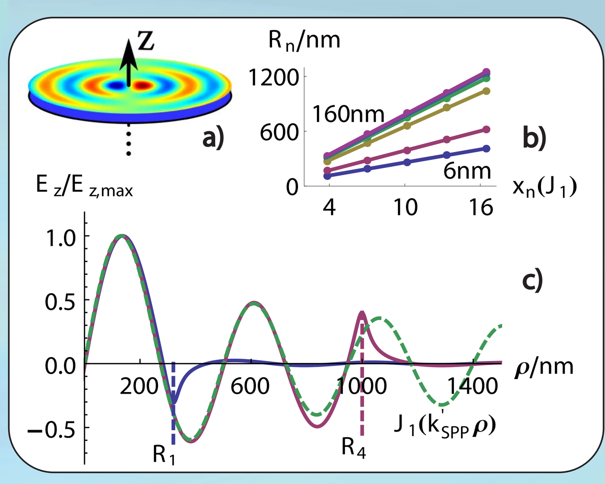
$$2\cdot k'_{SPP}R_{n,m} + \phi_m^r = 2\cdot x_n(J_m)$$



A plane wave excites a dipolar plasmonic Bessel-resonance of an 80 nm thick metallic disc. The reflection phase leads to an apparent length change.

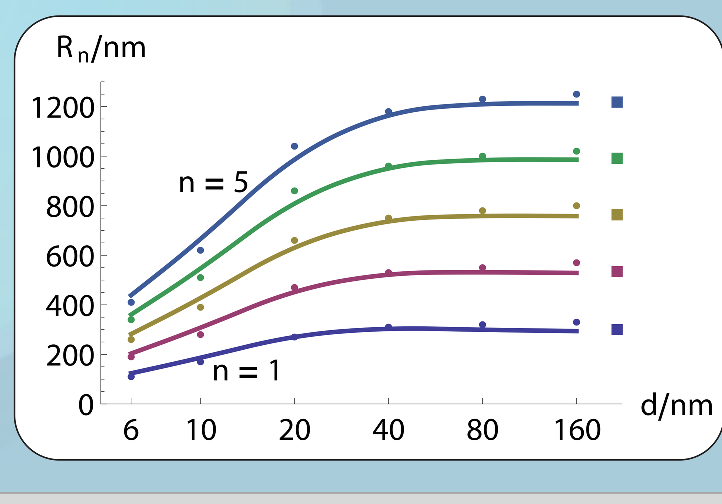
Field Profile and Scaling

- **resonator model** implies **form** of the field and **scaling** of resonant radii
  - educated guesses; have to be **verified**; simplest structure: a metallic disc
  - numerical **simulations**: plane wave excitation at ν = 625 Thz

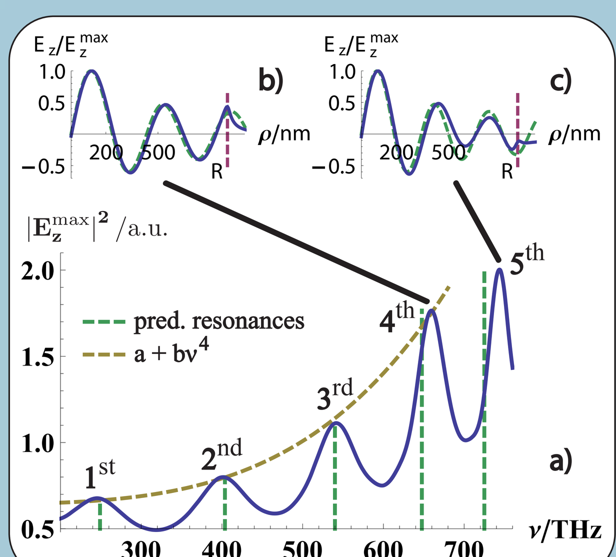


- **a) & c) field profile**
  - 80 nm thick disc, excitation of even mode
  - the electric field shows a **qualitative agreement** to a Bessel type plasmonic field
- **b) scaling**
  - resonant radii for thicknesses from 6 nm to 160 nm are linearly related to the roots of J_1

- **resonant radii**: **theory** (full lines) **vs.** full wave **simulations** (dots)
  - **resonance condition** with calculated phases of reflection **predict resonant** disc **radii** for different thicknesses d
  - **agreement for all** observed **orders** n

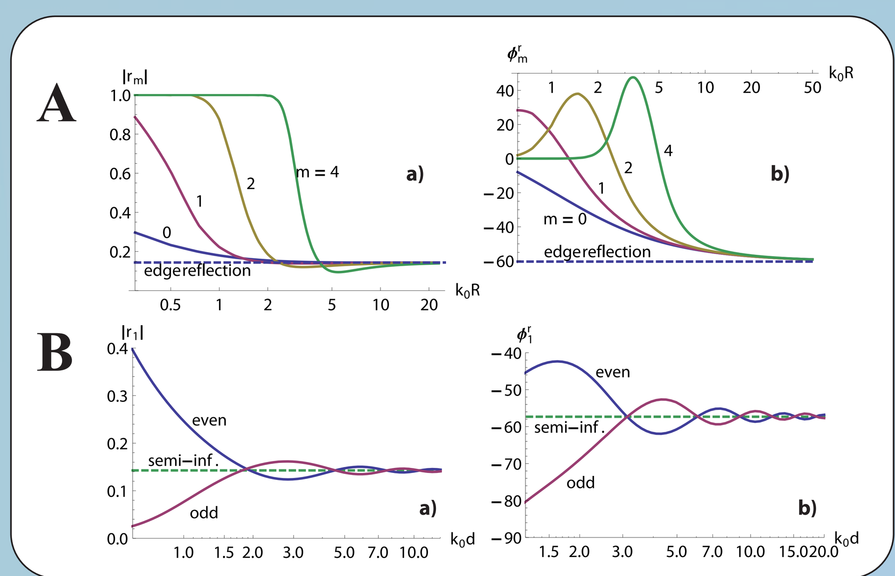


Spectral Predictions



- **resonances** of a silver disc, $R = 900$ nm
  - **a) spectrum** with predicted resonances
  - **b) & c) actual field distributions** for 4th and 5th resonance
- SPP highly **damped** for 5th order - deviation from Bessel form, other effects important
- also spectral **very good agreement** to numerics within limitations of theory

Limiting Cases



- **A**: **verification** of known results [4] in infinite disc limit for several orders
- **B**: **even and odd modes converge** to same result for increasing thickness

Conclusions

- **theory** for radially propagating Hankel-type SPPs in piecewise homogeneous **circular nanoantennas**
- **properties explained** by Fabry-Perot model using phase of reflection in agreement to simulations
- **antenna properties tunable** via stack composition

References

[1] P. Mühlschlegel et al. *Resonant optical antennas*, Science **308**, 1607 (2005)
[2] J. Dorfmüller et al. *Near-field Dynamics of Optical Yagi-Uda Nanoantennas*, Nano Lett. **11**, 2819 (2011)
[3] T. H. Taminiau et al. *Optical Nanorod Antennas Modeled as Cavities for Dipolar Emitters*, Nano Lett. **11**, 1020 (2011)
[4] R. Gordon *Vectorial method for calculating the Fresnel reflection of surface plasmon polaritons*, Phys. Rev. B **74**, 153417 (2006)

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